DESIGN AGAINST STATIC LOAD

DESIGN OF SIMPLE MACHINE PARTS
COTTER JOINT

Cotter and Knuckle joints are fasteners used to connect two roads transmitting axial force.

A cotter Joint is used to connect two co-axial rods, which are subjected to either axial tensile force or axial compressive force.

It is also used to connect a rod on one side with some machine part like a crosshead or base plate on the other side.

It is not used for connecting the shaft that rotate and transmit torque.

Applications:

- Joint between the piston rod and the cross head of a steam engine.
- Foundation bolt
- Joint between the piston rod and the tail or pump rod
COTTER JOINT

A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment.

The cotter is usually made of mild steel or wrought iron.

It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod, strap end of connecting rod etc.

Types of Cotter Joints

- Socket and spigot cotter joint,
- Sleeve and cotter joint, and
- Gib and cotter joint.
The cotter has uniform thickness and the width dimension is given a slight taper.

The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a locking device is provided.

- The taper is provided for the following reason.
  - When the cotter is inserted in the slot through the socket and the spigot and pressed by means of hammer, it becomes tight due to wedge action. This ensures tightness of the joint in operating condition and prevents loosening of the parts.
  - Due to its taper shape, it is easy to remove the cotter and dismantle the joint.
- Machining a taper on two sides of a machine part is more difficult than making a taper on one side. Also, there is no specific advantage of a taper on two sides.
A clearance of 1.5 to 3 mm is provided between the slots and the cotter for the adjustment.
The cotter joint offers the following advantages:

- The assembly and dismantling of parts of the cotter joint is quick and simple.
- The assembly consists of inserting the spigot end into the socket end and putting the cotter into their common slot.
- When the cotter is hammered, the rods are drawn together *and* tightened. Dismantling consists of removing the cotter from the slot by means of a hammer.
- The wedge action develops a very high tightening force, which prevents loosening of parts in service.
- The joint is simple to design and manufacture.
DESIGN OF SOCKET AND SPIGOT COTTER JOINT

- **P** = tensile force acting on rods (N)
- **d** = diameter of each rod (mm)
- **d₁** = outside diameter of socket (mm)
- **d₂** = diameter of spigot or inside diameter of socket (mm)
- **d₃** = diameter of spigot-collar (mm)
- **d₄** = diameter of socket-collar (mm)
- **a** = distance from end of slot to the end of spigot on rod-B (mm)
- **b** = mean width of cotter (mm)
- **c** = axial distance from slot to end of socket collar (mm)
- **t** = thickness of cotter (mm)
- **t₁** = thickness of spigot-collar (mm)
- **l** = length of cotter (mm)
DESIGN OF SOCKET AND SPIGOT COTTER JOINT

\( \sigma_t = \text{Permissible tensile stress for the rods material}, \)
\( \tau = \text{Permissible shear stress for the cotter material}, \) and
\( \sigma_c = \text{Permissible crushing stress for the cotter material}. \)

Following assumptions are made:

- The rods are subjected to axial tensile force.
- The effect of stress concentration due to the slot is neglected.
- The stresses due to initial tightening of the cotter are neglected.

- In order to design the cotter joint and find out the dimensions, failures in different parts and at different cross-sections are considered.
- Based on each type of failure, one strength equation is written and these equation is used to determine the various dimensions of the cotter joint.
Failure of the rods in tension:

Each rod of diameter $d$ is subjected to a tensile force $P$.

$$resting\ Area = \frac{\pi}{4} d^2$$

$$\sigma_t = \frac{P}{\frac{\pi}{4} d^2} \text{ or } \frac{P}{\pi d^2}$$

$$d = \sqrt{\frac{4P}{\pi \sigma_t}}$$
FAILURE OF SPIGOT IN TENSION ACROSS THE WEAKEST SECTION (OR SLOT)
Failure of spigot in tension across the weakest section (or slot)

The weakest section of the spigot is that section which has a slot in it for the cotter.

Resisting Area = \( \frac{\pi}{4} (d_2)^2 - d_2 \times t \)

\[ \sigma_t = \frac{P}{\frac{\pi}{4} (d_2)^2 - d_2 \times t} \]

\[ P = \left[ \frac{\pi}{4} (d_2)^2 - d_2 \times t \right] \sigma_t \]

where, thickness of cotter \( t = 0.31 \, d \)

From this equation, the diameter of spigot or inside diameter of socket \((d_2)\) may be determined.
TENSILE FAILURE OF THE SOCKET ACROSS THE SLOT
Tensile Failure of the socket across the slot

resisting Area \[ A = \frac{\pi}{4} \left[ (d_1)^2 - (d_2)^2 \right] - (d_1 - d_2) t \]

tensile strength \[ \sigma_t = \frac{P}{\frac{\pi}{4} \left[ (d_1)^2 - (d_2)^2 \right] - (d_1 - d_2) t} \]

\[ P = \left\{ \frac{\pi}{4} \left[ (d_1)^2 - (d_2)^2 \right] - (d_1 - d_2) t \right\} \sigma_t \]

From this equation, outside diameter of socket \((d_1)\) may be determined.
shear failure of cotter

From this equation, width of cotter ($b$) is determined.

where, $\tau = \text{Permissible shear stress for the cotter material}$

$$P = 2bt \times \tau$$

$$\tau = \frac{P}{2bt}$$

$$b = \frac{P}{2\tau t}$$
SHEAR FAILURE OF SPIGOT END
Shear Failure of Spigot end

Spigot end is in double shear.

The area of each of the two planes that resist shear failure is

$$\tau = \frac{P}{2a d_2}$$

where, $\tau$ = Permissible shear stress for the Spigot material.

From this equation, the distance from the end of the slot to the end of the Spigot ($a$) may be obtained.
Shear Failure of Socket end
Shear Failure of Socket end

Socket end is in double shear

\[ \tau = \frac{P}{2 (d_4 - d_2) c} \]

\[ P = 2 (d_4 - d_2) c \times \tau \]

Where, \( d_4 = 2.4 \, d \)

From this equation, Distance \((c)\) may be obtained.
Crushing Failure of Spigot End.

As shown in Fig, the force $P$ causes compressive stress on a narrow rectangular area of thickness $t$ and width $d_2$.

$$\sigma_c = \frac{P}{td_2}$$
Crushing Failure of Socket End or Socket Collar

From this Eq. diameter $d_4$ can be obtained.

$$\sigma_c = \frac{P}{(d_4 - d_2) t}$$
Failure of cotter in bending

\[ x = \frac{1}{3}, \quad y = \frac{1}{3} \left( \frac{d_4 - d_2}{2} \right) = \left( \frac{d_4 - d_2}{6} \right) \]

\[ M_b = \frac{P}{2} \left[ \frac{d_2}{2} + x \right] - \frac{P}{2} (z) \]

\[ = \frac{P}{2} \left[ \frac{d_2}{2} + \frac{d_4 - d_2}{6} \right] - \frac{P}{2} \left[ \frac{d_2}{4} \right] \]

\[ = \frac{P}{2} \left[ \frac{d_2}{4} + \frac{d_4 - d_2}{6} \right] \]

Also, \[ I = \frac{tb^3}{12} \]

and \[ \sigma_b = \frac{M_b y}{I} \]

Therefore,

\[ \sigma_b = \frac{P}{2} \left[ \frac{d_2}{4} + \frac{d_4 - d_2}{6} \right] \frac{b}{2} \left( \frac{tb^3}{12} \right) = \frac{P}{2 t \times b^2} \left( d_4 + 0.5 d_2 \right) \]
Empirical Relations

- $d_1 = 1.75d$
- $d_2 = 1.21d$
- $d_3 = 1.5d$
- $d_4 = 2.4d$
- $a = c = 0.75d$
- $b = 1.6d$
- $t = 0.31d$
- $t_1 = 0.45d$
- Taper of cotter 1 in 32
Ex. 1:-

Design a cotter joint to connect two steel rods of equal diameter. Each rod is subjected to an axial tensile force of 50 kN. Design the joint and specify its main dimensions.

- **Selection of material**  Plain Carbon Steel 30C8 (Syt= 400 N/mm²)
- **Selection of Factor of Safety**  (6 for rods, spigot, socket) (4 for cotter)
- **Calculation of Permissible Stresses**
  - Assumption:- Yield strength in compression is twice the yield strength in tension. Yield strength in shear is half the yield strength in tension.
- **Calculation of Dimensions**

  - Ans:-
  - \( d = 30.90 \text{ mm } = 32 \text{ mm} \)
  - \( d_1 = 55 \text{ mm} \)
  - \( d_2 = 40 \text{ mm} \)
  - \( d_3 = 48 \text{ mm} \)
  - \( d_4 = 80 \text{ mm} \)
  - \( t = 10 \text{ mm} \)
  - \( a = c = 20 \text{ mm} \)
  - \( b = 50 \text{ mm} \)
  - \( t_1 = 15 \text{ mm} \)
PROCEDURE

(i) Calculate the diameter of each rod by Eq.

(ii) Calculate the thickness of the cotter by the empirical relationship given in Eq.

(iii) Calculate the diameter $d_2$ of the spigot on the basis of tensile stress. From Eq.

(iv) Calculate the outside diameter $d_1$ of the socket on the basis of tensile stress in the socket, from Eq.
(v) The diameter of the spigot collar \( d_3 \) and the diameter of the socket collar \( d_4 \) are calculated by the following empirical relationships,

\[
\begin{align*}
    d_3 &= 1.5 \ d \\
    d_4 &= 2.4 \ d
\end{align*}
\]

(vi) The dimensions \( a \) and \( c \) are calculated by the following empirical relationship,

\[
a = c = 0.75 \ d
\]

(vii) Calculate the width \( b \) of the cotter by shear consideration using and bending consideration using Eq. and select the width, whichever is maximum between these two values

\[
b = \frac{P}{2 \tau t} \quad \text{or} \quad b = \sqrt{\frac{3P}{t \sigma_b} \left[ \frac{d_2}{4} + \frac{d_4 - d_2}{6} \right]}
\]
(viii) Check the crushing and shear stresses in the spigot end by Eqs. respectively.

\[
\sigma_c = \frac{P}{td_2}
\]

\[
\tau = \frac{P}{2ad_2}
\]

(ix) Check the crushing and shear stresses in the socket end by Eqs respectively.

\[
\sigma_c = \frac{P}{(d_4 - d_2) t}
\]

\[
\tau = \frac{P}{2 (d_4 - d_2) c}
\]
(x) Calculate the thickness \( t_1 \) of the spigot collar by the following empirical relationship, \( t_1 = 0.45 \, d \)

The taper of the cotter is 1 in 32.
Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically.

Tensile stress = 50 MPa; shear stress = 35 MPa and crushing stress = 90 MPa.

Given Data:-
\[ P = 30 \text{ kN} = 30 \times 10^3 \text{ N} \]
\[ \sigma_t = 50 \text{ MPa} = 50 \text{ N/mm}^2 \]
\[ \tau = 35 \text{ MPa} = 35 \text{ N/mm}^2 \]
\[ \sigma_c = 90 \text{ MPa} = 90 \frac{\text{N}}{\text{mm}^2} \]
# Design Procedure for Cotter Joint

Calculate the diameter of each rod:

\[ d = \sqrt{\frac{4P}{\pi \sigma_i}} \]

Calculate the thickness of the cotter by empirical relation:

\[ t = 0.31d \]

Calculate the diameter \( d_2 \) of the spigot on the basis of tensile stress:

\[ P = \left[ \frac{\pi}{4} (d_2)^2 - d_2 \times t \right] \sigma_i \]

Calculate the outside diameter \( d_1 \) of the socket on the basis of tensile stress:

\[ P = \left\{ \frac{\pi}{4} \left[ (d_1)^2 - (d_2)^2 \right] - (d_1 - d_2) \times t \right\} \sigma_i \]

Diameter of the spigot collar \( d_3 \) and the diameter of the socket collar \( d_4 \) are calculated by empirical relationships:

\[ d_3 = 1.5d \quad d_4 = 2.4d \]

Dimension \( a \) and \( c \) are calculated from following equation:

\[ P = 2ad_2 \times \tau \quad P = 2(d_4 - d_2)c \times \tau \]

Calculate the width \( b \) of the cotter by shear consideration and bending consideration and select width, whichever is maximum between these two values:

\[ b = \frac{P}{2\tau t} \quad \text{or} \quad b = \sqrt{\frac{3P}{t\sigma_b} \left[ \frac{d_2}{4} + \frac{d_4 - d_2}{6} \right]} \]

Check the crushing and shear stresses in spigot end:

\[ \sigma_c = \frac{P}{td_2} \quad \tau = \frac{P}{2ad_2} \]

Check the crushing and shear stresses in socket end:

\[ \tau = \frac{P}{2(d_4 - d_2)c} \quad \sigma_c = \frac{P}{(d_4 - d_2)t} \]

Calculate the thickness \( t_1 \) of the spigot collar by the empirical relation:

\[ t_1 = 0.45d \]

Taper of the cotter is 1 in 32

Length of Cotter:

\[ L = 4d \]
• Failure of spigot collar in crushing

\[ \sigma_c = \frac{\pi}{4} \frac{P}{(d_3^2 - d_2^2)} \]

From this diameter \(d_3\) can be obtained.

Crushing Failure of Socket End or Socket Collar

\[ \sigma_c = \frac{P}{(d_4 - d_2) t} \]

From this Eq. diameter \(d_4\) can be obtained.

Failure of the spigot collar in shearing

\[ \tau = \frac{P}{\pi d_2 t_1} \]

From this equation, the thickness of spigot collar \((t_1)\) may be obtained.
1. Diameter of the rods

Let \( d = \text{Diameter of the rods} \).

Considering the failure of the rod in tension. We know that load \((P)\),

\[
30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3 \ d^2
\]

\[
\therefore \quad d^2 = \frac{30 \times 10^3}{39.3} = 763 \quad \text{or} \quad d = 27.6 \text{ say 28 mm Ans.}
\]

Diameter of socket collar

Let \( d_4 = \text{Diameter of socket collar} \).

Considering the failure of the socket collar and cotter in crushing. We know that load \((P)\),

\[
30 \times 10^3 = (d_4 - d_2) \times t \times \sigma_c = (d_4 - 35) \times 10 \times 90 = (d_4 - 35) \times 900
\]

\[
\therefore \quad d_4 - 35 = \frac{30 \times 10^3}{900} = 33.3 \quad \text{or} \quad d_4 = 33.3 + 35 = 68.33 \text{ say 70 mm Ans.}
\]

Thickness of socket collar

Let \( c = \text{Thickness of socket collar} \).

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load \((P)\),

\[
30 \times 10^3 = 2(d_4 - d_2) \times c \times \tau = 2 (75 - 35) \times c \times 35 = 2800 \ c
\]

\[
\therefore \quad c = \frac{30 \times 10^3}{2800} = 12 \text{ mm Ans.}
\]
Diameter of spigot collar

Let \( d_3 \) = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load \((P)\),

\[
30 \times 10^3 = \frac{\pi}{4} \left[ (d_3)^2 - (d_2)^2 \right] \sigma_c = \frac{\pi}{4} \left[ (d_3)^2 - (35)^2 \right] 90
\]

or \( (d_3)^2 - (35)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424 \)

\[
(d_3)^2 = 424 + (35)^2 = 1649 \text{ or } d_3 = 42 \text{ mm} \text{ Ans.}
\]
Ex. 2:-

Two rods are connected by means of a cotter joint. The inside diameter of the socket and outside diameter of socket collar 50 and 100 mm respectively. The rods are subjected to a tensile force of 50 kN. The cotter is made of steel 30C8($s_{yt}=400$ N/mm$^2$) and the factor of safety is 4. the width of the cotter is five times of thickness.

Calculate:

• (1) Width and thickness of the cotter on the basis of shear failure; and
• (2) width and thickness of the cotter on the basis of bending failure.
Knuckle Joint
KNUCKLE JOINT

Two or more rods subjected to tensile forces are fastened together.

The joint allows a small angular moment of one rod relative to another.

It can be easily connected and disconnected.

Applications: Elevator chains, valve rods, link of a cycle chain, tie rod joint for roof truss etc.
KNUCKLE JOINT
KNUCKLE JOINT

In knuckle joint one end of the rods is made into an eye and the end of the other rod is formed into a fork with an eye in each of the fork leg.

The knuckle pin passes through both the eye hole and the fork holes and may be secured by means of a collar and taper pin or spilt pin.

The knuckle pin may be prevented from rotating in the fork by means of a small stop, pin, peg or snug.

The material used for the joint may be steel or wrought iron.
\[ P = \text{Tensile load acting on the rod,} \]
\[ D = \text{Diameter of each rod,} \]
\[ D_1 = \text{Enlarge Diameter of each rod,} \]
\[ d = \text{Diameter of Knuckle pin,} \]
\[ d_0 = \text{Outer diameter of eye or fork,} \]
\[ \mathbf{a} = \text{Thickness of single fork,} \]
\[ \mathbf{b} = \text{Thickness of eye end of rod - B.} \]
\[ \sigma_t, \tau \text{ and } \sigma_c = \text{Permissible stresses for the joint material in tension, shear and crushing respectively.} \]
\[ x = \text{distance of the centre of fork radius } R \text{ from the eye} \]
DESIGN PROCEDURE FOR KNUCKLE JOINT

In determining the strength of the joint for the various methods of failure, it is assumed that

1. There is no stress concentration, and
2. The load is uniformly distributed over each part of the joint.

Failure of the solid rod in tension

\[
\sigma_t = \frac{P}{\left(\frac{\pi}{4}D^2\right)} \quad D = \sqrt{\frac{4P}{\pi \sigma_t}} \quad D_1 = 1.1 \cdot D
\]
Failure of the knuckle pin in shear

\[ \tau = \frac{P}{2 \left( \frac{\pi}{4} d^2 \right)} \]

\[ d = \sqrt{\frac{2P}{\pi \tau}} \]

\[ d_1 = 1.5d \]

\[ a = 0.75D \]

\[ b = 1.25D \]
The bending moment is maximum at the centre. It is given by,

\[ M_b = \frac{P}{2} \left[ \frac{b}{2} + x \right] - \frac{P}{2} \left( z \right) \]

\[ = \frac{P}{2} \left[ \frac{b}{2} + \frac{a}{3} \right] - \frac{P}{2} \left[ \frac{b}{4} \right] \]

\[ = \frac{P}{2} \left[ \frac{b}{4} + \frac{a}{3} \right] \]

Also,

\[ I = \frac{\pi d^4}{64} \quad \text{and} \quad y = \frac{d}{2} \]

\[ \sigma_b = \frac{M_b y}{I} = \frac{P}{2} \left[ \frac{b}{4} + \frac{a}{3} \right] \frac{d}{2} \]

\[ = \frac{32}{\pi d^3} \times \frac{P}{2} \left[ \frac{b}{4} + \frac{a}{3} \right] \]
Crushing Failure of Pin in Eye

\[ \sigma_c = \frac{P}{b \times d} \]

\[ b = 1.25D \]
Crushing Failure of Pin in Fork

\[ \sigma_c = \frac{P}{2\pi ad} \]

\( a = 0.75D \)
Tensile Failure of Eye

The tensile stress at section XX is given by:

\[\sigma_t = \frac{P}{\text{area}}\]

\[\sigma_t = \frac{P}{b(d_o - d)}\]
Shear Failure of Eye

\[ \tau = \frac{P}{2b(d_o - d)/2} \]

\[ \tau = \frac{P}{2b(d_o - d)} \]
area = 2a (d_0 - d)

Tensile stress in the fork is given by,

\[ \sigma_t = \frac{P}{2a(d_0 - d)} \]
Shear Failure of Fork

\[ \tau = \frac{P}{2a(d_0 - d)} \]
DESIGN PROCEDURE

The basic procedure to determine the dimensions of the knuckle joint consists of the following steps:

- Calculate the diameter of each rod ‘D’
  \[ D = \sqrt{\frac{4P}{\pi\sigma}} \]

- Calculate the enlarge diameter of each rod by empirical relationship
  \[ D_1 = 1.1D \]

- Calculate the dimensions a and b by empirical relationship
  \[ a = 0.75D \]
  \[ b = 1.25D \]

- Calculate the diameters of the pin by shear consideration and bending consideration and select the diameter, whichever is maximum.
  \[ d = \sqrt{\frac{2P}{\pi\tau}} \text{ (or) } d = 3\sqrt{\frac{32}{\pi\sigma_b}} \times \frac{P}{2} \left[ \frac{b}{4} + \frac{a}{3} \right] \]

- Calculate the dimensions \( d_0 \) and \( d_1 \) by empirical relationships
  \[ d_0 = 2d \]
  \[ d_1 = 1.5d \]
Check the tensile, crushing and shear stresses in the eye

\[ \sigma_t = \frac{P}{b(d_o - d)} \]
\[ \sigma_c = \frac{P}{bd} \]
\[ \tau = \frac{P}{b(d_o - d)} \]

Check the tensile, crushing and shear stresses in the Fork

\[ \sigma_t = \frac{P}{2a(d_o - d)} \]
\[ \sigma_c = \frac{P}{2ad} \]
\[ \tau = \frac{P}{2a(d_o - d)} \]
EXAMPLE 1

It is required to design a Knuckle joint to connect two circular rods subjected to an axial tensile force of 50 kN. The rods are coaxial and a small amount of angular movement between their axis is permissible. Design the joint and specify the dimensions of its components.

Consider material for two rods and pin is Plain Carbon Steel 30C8 ($S_{yt}=400 \text{ N/mm}^2$) and the factor of Safety is 5

Assumption:- Yield strength in compression is equal to the yield strength in tension.