Unit-2 Circular Motion

★ Define:-

1) Force :- External effort in the form of push or pull which produces or attempts to produce motion in an object at rest. Stops or tries to stop the object in motion. Change or tries to change the direction of motion of motoin of an object is called force.

$$Force = Mass \times Acculeration$$

$$\therefore F = ma$$

SI unit = N   CGS unit = dyne

2) Momentum :- The momentum of a body is defined as the product of the mass of the body and its velocity

$$Momentum = mass \times velocity$$

$$\vec{p} = m\vec{v}$$

SI unit = $Kg \cdot ms^{-1}$ CGS unit = $gm\text{-}cms^{-1}$

3) Impluse of force : The product of force and time during which force acts is called impoluse of force.

Impluse of force = $F \cdot dt$

Impluse of force = Change in momentum

SI unit = newton-second (Ns) or $Kg\ ms^{-1}$

CGS unit = dyne x second

4) Circular motion : At a constant velocity, A body describing a circular path rotating about fixed axis said to have rotatory or circular motion.

5) Angular displacement : Angular displacement is defined as a angle between the initial and the final positions for a given object having a circular motion about a fixed point.

(.) Radian (θ) : At the center of a circle, an are with a radius of a circle forming an angle is called radian.

$$\theta = \frac{\text{length of arc}}{\text{radius of circle}} = \frac{s}{r}$$

Angle formed at the center $\theta = \frac{2\pi r}{r} = 2\pi$ radian.

$1\ rad = 360/(2\pi) = 57°3$

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6) Angular velocity :- The angle described by a rotating body per unit time is called its angular velocity and is denoted by the ω (omega) Angular velocity of the body.

$$\omega = \frac{\text{Angular displacement}}{\text{time}} = \frac{\theta}{t}$$

SI Unit = $\frac{rad}{s}$

7) Linear velocity :- Let us assume that the body travels on circular path within uniform speed v. If the angle described is θ and the distance traversed is S in time t i.e., reached at A to B.

$$\text{Speed } v = \frac{\text{distance}}{\text{time}} = \frac{S}{t}$$

$v = \frac{r\theta}{t}$ ($\because S = r\theta$) θ = angular displacement
S = (length of arc)

$\theta t = \omega$

$\therefore V = r\omega$ ($\because \theta/t = \omega$) ω = angular velocity
r = radius
v = linear velocity

8) Angular acceleration:- The angular acceleration of a rotating body is defined as the rate of change of angular velocity.

α = Angular acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t} \ rad/s^2$$

9) Centripetal force:- The force exterted by the radius towards the center of a circle which makes an object move in a circle path with uniform speed is called centripetal force.

(o) Centripetal acceleration:- Change in velocity due to the centripetal force is called centripetal acceleration.

10) Centrifugal force:- The outward pushing force on an object in a circular motion called centrifugal force.

$$F_c = \frac{mv^2}{r}$$

☆ Relation Between Angular velocity and periodic time

$$\therefore \omega = \frac{2\pi}{T} \qquad \therefore \frac{1}{T} = f$$

Where ω = omega
T = periodic time
f = frequency

☆ Relation Between Angular velocity and frequency

$$\therefore \omega = 2\pi f$$

Where ω = omega
f = frequency

☆ Relation Between Angular velocity and leniar velocity

Let us assume that the body travels on circular path with unifrom speed v. If the angle descirbed is θ and the distance transversed is S in time i.e reached at A to B.

$$\text{Speed } v = \frac{\text{distance}}{\text{time}} = \frac{S}{t}$$

$$V = \frac{r\theta}{t} \quad (\because S = r\theta)$$

θ = angular displacement
S = length of arc

$$\therefore V = r\omega \quad (\because \theta/t = \omega)$$

ω = angular velocity
r = radius
V = linear velocity

Relation Between Angular ~~accrl~~ acculeration and linear acculeration

The angular acculeration of a rotating body is defined as the rate of change of angular velocity

α = Angular acculeration

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t} \text{ rad}/s^2$$

liner acculeration of the body will be

$$a = r\frac{d\omega}{dt}$$

a = linear acculeration

linear acculeration = distance from the axis of rotation × angular acculeration

$$\therefore a = r\alpha$$

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☆ Answer the following question in Brief :-

1.) Law of conservation of momentum :-

Statement : The total momentum of an isolated system remains constant.

Explanation with illustration :

Let us considered an example of a bullet fired from a rifle. As the bullet fired from the rifle moves forward, the rifle is pushed backward. If the force on the bullet by the rifle is F, then the force exerted on the rifle by the bullet is $\vec{F}$. Both these forces act for the same time interval Δt. Before the bullet is fired both the bullet and rifle were steady.

Therefore their respective momentum $\vec{pb}$ and $\vec{pr}$ both were zero. Hence their total initial momentum.

$$\vec{Pb} + \vec{Pr} = 0 \ldots\ldots (1)$$

Now as we Known
Change in momentum of bullet = $\vec{F}\Delta t$ ($dp = Fdt$)
Change in momentum of rifle = $-\vec{F}\Delta t$

As initial momentum of each of them is zero final momenta will be equal to the change in respective momenta.

Thus
Final momenta of bullet = $\vec{P'b} = F\Delta t$
and Final momenta of rifle = $\vec{P'r} = -F\Delta t$

Adding two results

$$P'b + P'r = F\Delta t - F\Delta t$$
$$P'b + P'r = 0 \text{ ---- } (2)$$

From relation (1) & (2)

$$P'b + P'r = Pb + Pr$$

i.e. [final momentum of (bullet + rifle)] = [inital momentum of (bullet + rifle)]

2) Define impluse of force and state its application

The product of force and time during which forces acts is called impulse of force

$$\text{Impulse of force} = F \cdot dt$$

$$F = \frac{dp}{dt}$$

$$\text{Impulse of force} = \text{change in momentum}$$

SI unit = newton-second (Ns) or $Kg\, ms^{-1}$

CGS unit = dyne x second or gm-cm/s

(•) Applications of impulse of force:-

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(1) A cricket player lowers his hands while catching a cricket ball. As you may have noticed, the players draws in his allows a longer time for his hands to stop the ball. Therefore by increasing the time of a catch, the player has to apply a smaller force against the ball in order to stop it. The ball in turn, exerts a smaller force on his hands and his hands are not injured.

(2) When a person falls from a certain height on a cemented floor, the floor does not yield. The total change in linear momentum is produced in a smaller interval of time. Therefore, as explained above, the floor ~~exterts~~ exerts a much large force. Due to it, a person gets more injury.

3) Chinawares and glasswares are wrapped in paper or straw pieces while packing. In the event of fall, impact will take a longer time to reach the glass/chinawares through paper/straw. As a result, the average force exerted on the china or glasswares is small and chances of their breaking reduce.

4) The vehicles like scooter, car, bus, truck etc. are provided with shock-absorber when they move over an uneven road, impulsive forces are exerted by the rod. The function of shock-absorber is to increase the time of impact. This would reduce the force/jerk experinced by the rider of the vehicle.

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5) Bogies of a train are provided with the buffers They avoid severe jerks during shunting of the train.

(.) Due to presence of buffers, time of impact increases. Therefore, force during jerks decrease. Hence the chances.

☆ give difference between centripetal force and centrifugal force

Centripetal force	Centrifugal force
The force exerted by the radius towards the center of a circle which makes an object move in a circle path with uniform speed is called centripetal force	The outward pushing force on an object in a circular motion called centrifugal force.

★ State the example of centrifugal and centripetal force:-

1) The force of fricition between the road and the tire provides the desired centripetal force when the motor car moves on a circular road. This fricitional force balances the centrifugal force. Fricitional seens to be less if the road is too smooth. As a result, the motor car skids off. For this the ~~dinner~~ inner edge of the road is kept slightly lower than the outer edge. Thus curved roads are made sloping.

2) Mudgard in bicycle (mud guard): When a bicycle is moving on the road, mud, dust etc. stick the tire and it flies tangentailly as the speed increases. As a result, the rider's clothes get spoil. That's why Mudguards are kept on the tires.

3) Bending of cyclists at curved roads are also examples of centripetal force and centrifugal force.

★ Short note:-

(1) Recoiling of a gun:-

Explanation with illustration:

Let us considired an example of a bullet fired from a rifle. As the bullet fired from the rifle moves forward, the rifle is pushed backword. It the force on the bullet by the rifle is $\vec{F}$, then the force exerted on the rifle by the bullet is $-\vec{F}$. Both these forces act for the same time interval Δt. Before the bullet is fired both the bullet and rifle were steady.

Therefore their respective momenta $\vec{P_b}$ and $\vec{P_R}$ both were zero. Hence their total intial momentum.

$$\vec{P_b} + \vec{P_R} = 0 \ldots\ldots (1)$$

Now as we Known
Change in momentum of bullet = $\vec{F}\,\Delta t$ ($dp = Fdt$)
Change in momentum of rifle = $-\vec{F}\,\Delta t$

As intial momentum of each of them is zero final momenta will be equal to the change in respective momenta.

Thus
Final momenta of bullet = $\vec{P'b}$ = $F\Delta t$
and final momenta of rifle = $P'r$ = $-F\Delta t$

Adding two results

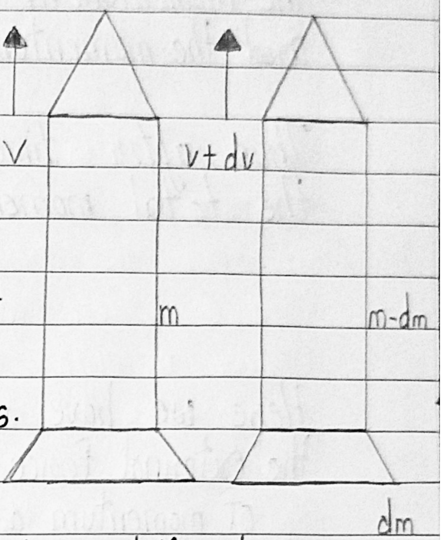
$$P'b + P'r = F\Delta t - F\Delta t$$
$$P'b + P'r = 0 \ldots\ldots (2)$$

From relation (1) & (2)

$$P'b + P'r = Pb + Pr$$

i.e. $\left[\begin{array}{c}\text{final momentum} \\ \text{of (bullet + rifle}\end{array}\right] = \left[\begin{array}{c}\text{intial momentum} \\ \text{of (bullet + rifle)}\end{array}\right]$

(2) Rocket populsion :- For instance in a rocket, the gases produced by the combustion of fuel, stream out at great speed giving a thrust to the rocket in the opposite direction. If the rocket is identifed as the system mass of the system is obviosly not constant; it decreases continuosly with time.



The gas can be considered as coming out constant velocity w.r.t the rocket and hence the rate of gas be considered as constant. In this case eventhough the force on rocket is constant its acceleration goes increasing with time because of continous reduction in its mass.

The apparatus put in a rocket to perform differnt experiments in space are collectively Known as played. We take upward direction as the direction of motion of rocket. Considering the rocket and fuel in it as a system, we denote the mass and the velocity of the system, at any time t, as M and Vr respectively. Here the velocity is taken w.r.t some stationary be Vg. The velocity of the gas coming out and as a result the velocity imparted to the rocket are in mutually opposite directions. Hence the velocity of the gas, w.r.t rocket, is denote by -VgK. the velocity of the gas, w.r.t the stationary reference frame = Vr - Vg.

At any time t the momentum of rocket = mVr ... (1)

In an infinitestesimally small time interval dt, after t, if the decrease in the mass of the rocket is dm, so mass of the rocket after time dt is $m - dm$ and the velocity of the rocket is $V_r + dV_r$.

The momentum of rocket at time $(t + dt) = (m - dm)(V_r + dV_r)$ and the momentum of the gas ejected $= dm\,(V_r - V_g)$

Thus, after time interval dt the total momentum of the whole system $(m-dm)(V_r+dV_r)$

$$+ (dm\,(V_r - V_g))$$
$$= mV_r + m\,dV_r - dm\,V_g$$

Here we have neglected the term as it is very small. Since the external force on the system can use the law of conservation of momentum and write

$$MV_r = mV_r + m\,dV_r - dm\,V_g$$
$$dm\,V_g = m\,dV_r$$

$$-V_g \frac{dm}{dt} = m \frac{dV_r}{dt} \quad \ldots\ldots (2)$$

Since the mass decreases with time, we have put negative sign here

Here, $\frac{dV_r}{dt} = a$ is the acculiration of the rocket and $m\frac{dV_r}{dt}$

is the force (F) on the rocket. So the term is the force acting on the rocket, in the direction of motion of the rocket and

$V_g \frac{dm}{dt} = F$ is called the thrust on the rocket.

To increase the thrust on the rocket, ~~in the direction~~ of either the velocity v of the gas ejected from the rocket must be increase of lose of mass of the rocket should be increase.

At the time of launching a rocket, the weight of the rocket and the resistant air should also be considered. At the time of launching, the thrust on a rocket should more than its weight.

$$-\frac{dm}{m} = \frac{dv_r}{V_g}$$

When the intial mass is m_0, its velocity is zero. Denoting the velocity rocket as V when its mass is m and integrating eqn (2). We get

$$V = V_g \ln\left(\frac{m_0}{m}\right) \dots \quad (3)$$

The initial mass of a rocket is $m_0 = m_{payload} + {}_{body} + {}^{m}{}_{fuel}$. When the fuel in the rocket gets burnt out complete the mass of the rocket is $m = m_{payload} + {}_{body}$ and at the time the velocity of the rocket would be maximum which can be calculate as follows

$$V(max) = V_g \ln\left(\frac{m_{fuel} + m_{payload} + m_{body}}{m_{payload} + m_{body}}\right)$$

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$$V(max) = V_g \ln\left(\frac{m_{fuel}}{m_{payload} + m_{body}} - 1\right)$$

It can be seen from this equation that in order to increase the magnitude of maximum velocity–

★ Banking of curved roads:–

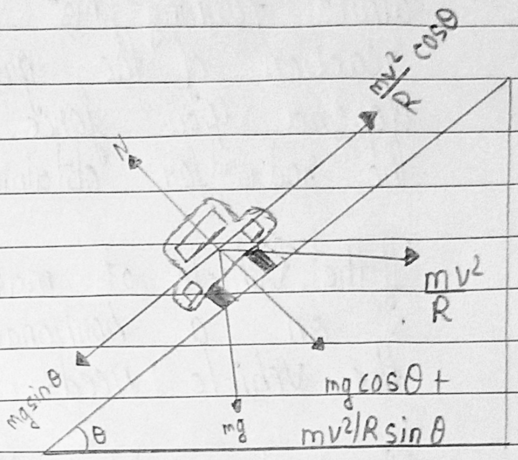
When a car goes round a level curve the necessary centripetal force is provided by the force of friction between the tyres and the road. In case the friction force, which acts as centripetal force and keeps the body moving along the circular road path, is not enough to provide the necessary centripetal force, the car will skid. To avoid the skidding of the car the rod is blanked in such a way that the outer part of the road is raised a little above the inner side, which makes the road track sloping towards the centre of the curve as shown in fig The inclination of the road bed with the horizontal is chosen in such a way that the horizontal component of the normal reaction provides the centripetal forces and the vertical component of the reaction supports the weight.

In fig a car of mass m is traveling on a circular banked road. Let θ be the angle of banking, v be the velocity of the car and R be the normal reaction. The centripetal force is provided by the horizontal component of reaction $R\sin\theta$ and weight mg of the car is supported by the vertical component of $R\cos\theta$. Thus

We can have a formula for angle of banking as below

Component of centripetal force = Component of mass

$$\frac{mV^2}{R}\cos\theta = mg\tan\theta$$



$$\frac{mV^2}{R} = mg\tan\theta$$

$$\frac{\frac{mV^2}{R}}{mg} = \tan\theta$$

$$\frac{V^2}{Rg} = \tan\theta$$

$$\tan\theta = \frac{V^2}{Rg}$$

Where, θ = Angle of banking
V = speed of vehicle
R = radius of curved road
g = acceleration due to gravity
$\frac{mV^2}{R}$ = centripetal force
m = mass of vehicle.

From above equation it is clear that the angle of banking depends upon the radius of the curve of the road and the speed of the vehicle.

☆ Bending of cyclist round a curve :-

When a cyclist takes a turn, he also requires some centripetal force. If he Keeps himself vertical while turning his weight is balanced the normal reaction of the ground. In that event, he was to depend upon force of fricition between the tyres and the ~~rod~~ road for obtaining the necessary centripetal force.

If the vehicle of mass m has run safty with velocity v on a horizonal circular path of radius r, then the vehicle needs centrifugal force $F_c = \frac{mv^2}{r}$

The fricition between the tire and the road

$F_m = \mu_s mg$ (here μ_s is the coefficient of fricition between the tire and the road)

If the velocity of the ~~coefficient~~ vehicle is v then

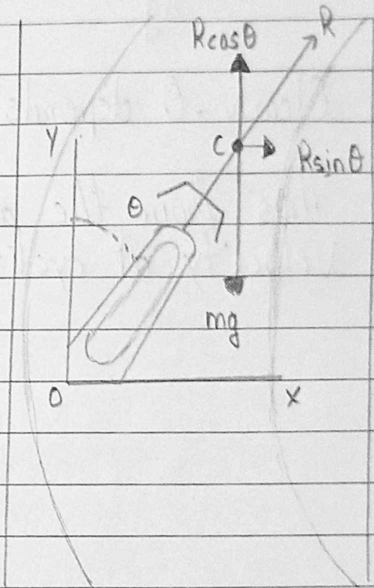
$\mu_s\, m\, g > mv^2/r$

For maximum safe velocity V_{max} of the vehicle

$$\frac{mV^2_{max}}{r} = \mu_s mg$$

$$V_{max} = \sqrt{\mu_s rg}$$

As force of friction is small and uncertain, dependence on it is not safe. To avoid dependence on force for obtaining centripetal force, the cyclist has to bend a little inwards from his vertical position, while turning By doing so a component of normal reaction in the horizontal direction provides the necessary centripetal force. To calculate the angle of bending with vertical



Suppose

m = mass of the cyclist
V = velocity of the cyclist while turning
r = radius of the circular path
θ = angle of bending with vertical

In fig. we have shown weight of the cyclist acting vertically downwards at the circle of gravity R is force of reaction of the ground on the cyclist. It acts at an angle θ which the vertical.

R can be resolved into two perpendicular components:

$R\cos\theta$, along the vertical upward direction

$R\sin\theta$, along the horizontal, towards the centre of the circular track

In equilibrium

$$R\cos\theta = mg \quad \dots (1)$$

and R Sin θ provides the necessary centripetal force

$R \sin \theta = \frac{mv^2}{r}$ ..... (2)

Dividing (2) by (1), we get

$\tan \theta = \frac{v^2}{rg}$

Clearly, θ depends on V and r

Thus from the above relation we see that the greater the velocity of cyclist and sharp the curve the move he must lena.